Syllabus

1. Course Description
   1. Title of a Course Modern Methods of Data Analysis: Stochastic Calculus
   2. Pre-requisites Probability theory, linear algebra, mathematical analysis.
   3. Course Type: core
   4. Abstract

The aim of this course is to provide an introduction to the modern methods of stochastic calculus. The course consists from two parts. The main emphasis of the first part will be in stochastic differential equations, their analytic and numerical solutions. We also briefly recall all necessary facts from the basic of random processes, Wiener process and Martingales.

The main emphasis of the second part will be on Markov chains. We discuss properties of Markov Chains, study their invariant distributions and convergence to stationary distributions. At the end of the course we discuss Markov Chain Monte-Carlo method (MCMC).

1. Learning Objectives

Students will study how to apply the main modern probabilistic methods in practice and learn important topics from the stochastic calculus.

1. Learning Outcomes

Know

1. Acquaintance with the main aspects of the stochastic analysis
2. Understand stochastic analysis and its applications in science and practice
3. Interrelation between different directions of modern stochastic analysis
4. How to apply stochastic analysis in science and practice

Be able

1. Ability to solve practical problems with methods from stochastic analysis
2. Apply Markov Chain Monte-Carlo methods
3. Select the most efficient probability methods to solve problems in science and practice
4. Ability to make an oral and written presentation
5. Ability to work with research literature on the modern stochastic analysis
6. Course Plan

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| --- | --- | --- | --- | --- |
| # | Topic / Theme | Annotated summary of topic / theme | Lectures | Seminars |
| 1 | Basic of probability theory | Conditional expectation and conditional probability; different types of convergence of random variables | 4 | 4 |
| 2 | Random processes | Filtration, trajectories, Kolmogorov’s theorem, finite dimensional distribution | 4 | 4 |
| 3 | Gaussian distribution, main properties | Characteristic function, tail-bounds, characterisation properties, covariance matrix | 4 | 4 |
| 4 | Wiener process | Definition, trajectories, Markov’s property, construction of Wiener’s process | 6 | 6 |
| 5 | Martingales | Definition, main properties, main inequalities | 4 | 4 |
| 6 | Ito’s integral | Simple function, Ito’s isometry, Ito’s formula | 6 | 6 |
| 7 | Stochastic differential equations | Existence, uniqueness, analytical and numerical solutions | 4 | 4 |
| 8 | Introduction to Markov Chains | Definition, examples, Markov’s property, Strong Markov’s properties, Class structure | 4 | 4 |
| 9 | Hitting times and absorption probabilities | Definition, main theorems | 4 | 4 |
| 10 | Recurrence and transience | Definition, main theorems, Recurrence and transience of random walks | 4 | 4 |
| 11 | Invariant distributions | Definition, main theorems, Time reversal | 4 | 4 |
| 12 | Convergence to equilibrium | Definition, main theorems, coupling, total variation distance | 6 | 6 |
| 13 | Ergodic theorem | Definition, main theorems, MCMC | 6 | 6 |
| 14 | Individual projects |  | 4 | 4 |
| SUM: |  |  | 64 | 64 |

1. Reading List
   1. Required
2. James Norris, Markov Chains, author’s personal webpage: <http://www.statslab.cam.ac.uk/~james/Markov/>
3. Paolo Baldi, Stochastic Calculus An Introduction Through Theory and Exercises, Springer, 2017. Read this book on SpringerLink via HSE network.
4. Douc, R., Moulines, E., Priouret, P., Soulier, P., Markov Chains, Springer, 2019, Read this book on SpringerLink via HSE network.
   1. Optional
5. Alain Durmus, Eric Moulines, High-dimensional Bayesian inference via the Unadjusted Langevin Algorithm, Arxiv: 1605.01559, https://arxiv.org/abs/1605.01559
6. Grading System: grade components - home assignments (40%) + individual project (20%) + written final exam (40%).
7. Guidelines for Knowledge Assessment

Home assignment: should be done in the form of a written report. The sample of the task structure:

* Title page
* A4 format
* Task solution

Examples of the homework see in [1], [3].

Individual project:

Examples of topics:

* Markov Chain Monte-Carlo
* Numerical solution of SDE
* Construction of Wiener’s process
* Concentration inequalities for Markov Chains
* Mixing time of Markov Chains

Requirements to the report: volume: 5-10 pages. Contents: statement of the problem; description of the current situation in this field c. problem solution; numerical simulations; conclusion; list of references.

Requirements to the presentation:  
Students should use LaTeX\PowerPoint to create a presentation generalising results of the individual project. The presentation must consist of 5-10 slides.

1. Methods of Instruction

Lectures, seminars, homework, individual project, exam

1. Special Equipment and Software Support (if required)

Python (Open source)