Syllabus

1. Course Description
   1. Title of a Course Modern random matrix theory
   2. Pre-requisites Probability theory, linear algebra, mathematical analysis.
   3. Course Type: elective
   4. Abstract

The aim of this course is to provide an introduction to asymptotic and non-asymptotic methods for the study of random structures in high dimension that arise in probability, statistics, computer science, and mathematics.  
One of the emphases is on the development of a common set of tools that has proved to be useful in a wide range of applications in different areas. Topics will include concentration of measure, Stein’s methods, suprema of random processes and etc.

Another main emphasis is on the application of these tools for the study of spectral statistics of random matrices, which are remarkable examples of random structures in high dimension and may be used as models for data, physical phenomena or within randomised computer algorithms.  
The topics of this course form an essential basis for work in the area of high dimensional data.

1. Learning Objectives

Students will study how to apply the main modern probabilistic methods in practice and learn important topics from the random matrix theory.

1. Learning Outcomes

Know

1. Acquaintance with the main aspects of the measure concentration phenomenon
2. Understand random matrix theory and its applications in science and practice
3. Interrelation between different directions of modern high-dimensional probability theory
4. How to apply the main measure concentration inequalities in science and practice

Be able

1. Ability to solve practical problems with methods from modern probability and random matrix theory
2. Compute and estimate spectral statistics of random matrices from different random matrix ensembles
3. Select the most efficient probability methods to solve problems in science and practice
4. Ability to make an oral and written presentation
5. Ability to work with research literature on the modern probability theory
6. Course Plan

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| --- | --- | --- | --- | --- |
| # | Topic / Theme | Annotated summary of topic / theme | Lectures | Seminars |
| 1 | Concentration of measure phenomenon | Sub-Gaussian and sub-exponential distributions, concentration inequalities for sums of random variables, Bernstein's inequality | 4 | 4 |
| 2 | Random vectors in high dimension | Multivariate Gaussian distribution, distribution of norm of random vector, dimensionality reduction, Johnson-Lindenstrauss lemma | 4 | 4 |
| 3 | Random matrices in science and applications | Random matrices in statistics, physics, telecommunications, numerical analysis, community detection in networks | 2 | 2 |
| 4 | Norms of random matrices | Norm of a random symmetric matrix, norms of rectangular matrices, the moment method, Gaussian processes, Sudakov-Fernique inequality | 4 | 4 |
| 5 | Sums of random matrices | Matrix inequalities, matrix Bernstein inequality, Ahlswede–Winter Bound | 4 | 4 |
| 6 | Sample covariance matrices | Concentration inequalities and moment inequalities for the sample covariance matrices, spectral projectors, principal component analysis | 6 | 6 |
| 7 | Gaussian ensembles of random matrices | Gaussian Unitary Ensemble (GUE), Gaussian Orthogonal ensemble (GOE), Wishart ensemble, eigenvalues density, eigenvectors, Determinantal structure, Spectral statistics, Wigner-Dyson-Gaudin- Mehta conjecture | 4 | 4 |
| 8 | Limit theorems for spectra of random matrices | Stieltjes transform, Wigner’s semicircle law, Marchenko-Pastur law, local limit theorems, Stein’s method | 4 | 4 |
| 9 | Individual projects |  | 3 | 3 |
| SUM: |  |  | 32 | 32 |

1. Reading List
   1. Required
2. R. Veshynin, High dimensional probability, author’s personal website: https://www.math.uci.edu/~rvershyn/papers/HDP-book/HDP-book.html, 2019
3. R. Van Handel In Convexity and Concentration (Carlen et al., eds.), IMA Vol. 161, Springer, 2017, pp. 107-165, author’s personal website: https://[web.math.princeton.edu/~rvan/ima160525.pdf](http://web.math.princeton.edu/~rvan/ima160525.pdf)
4. T. Tao, Topics in Random Matrix Theory, author’s personal website: <https://terrytao.files.wordpress.com/2011/02/matrix-book.pdf>
   1. Optional
5. F. Gotze, A. Naumov, A. Tikhomirov, D. Timushev, On the local semicircular law for Wigner ensembles, 2018, Bernoulli, 24, 3 p. 2358-2400
6. Grading System: grade components - home assignments (40%) + individual project (20%) + oral final exam (40%).
7. Guidelines for Knowledge Assessment

Home assignment: should be done in the form of a written report. The sample of the task structure:

* title page
* A4 format
* Task solution

Examples of the homework see in [1], [3].

Individual project:

Examples of topics:

* Dimensionality reduction and Johnson-Lindenstrauss lemma
* Extreme singular values of rectangular random matrices and application in the community detection in   
  networks
* Financial applications of random matrices
* Products of random matrices in telecommunication
* Wigner-Dyson-Gaudin-Mehta conjecture
* Sample covariance matrices and principal component analysis
* Random matrix techniques in quantum information theory
* Power grid applications of random matrices

Requirements to the report: volume: 5-10 pages. Contents: statement of the problem; description of the current situation in this field c. problem solution; numerical simulations; conclusion; list of references.

Requirements to the presentation:  
Students should use LaTeX\PowerPoint to create a presentation generalising results of the individual project. The presentation must consist of 5-10 slides.

1. Methods of Instruction

Lectures, seminars, homework, individual project

1. Special Equipment and Software Support (if required)

Python (Open source)