**Программа учебной дисциплины «Временные ряды и случайные процессы (преподается на английском языке)»**

Утверждена

Академическим советом ООП

Протокол № от «\_\_»\_\_\_\_\_20\_\_ г.

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| --- | --- |
| Автор  | Горяинова Е.Р. |
| Число кредитов  | 4 |
| Контактная работа (час.)  | 46 |
| Самостоятельная работа (час.)  | 106 |
| Курс  | 4 |
| Формат изучения дисциплины | без использования онлайн курса |

1. Course Description

a. Title of a Course: Time Series Analysis and Stochastic Processes

b. Pre-requisites: basic courses in Calculus, Theory of Probability and Mathematical Statistics.

c. Course Type (compulsory, elective, optional): compulsory

d. Abstract

This course presents an introduction to time series analysis and stochastic processes and their applications in operations research and management science. Time series analysis includes Moving average models MA(q), Autoregressive models AR(p), Autoregressive-moving average ARMA(p; q) models, GARCH models and Vector autoregressive models. Stochastic processes will initially be discussed on basic processes and random walks. Subsequently, Markov chains, Poisson processes and Markov processes will be presented. Also Problems from queuing theory will be analyzed.

2. Learning Objectives: To familiarize students with the concepts, models and statements of the theory of time series analysis and stochastic processes.

3. Learning Outcomes:

• Know basics of time series analysis and stochastic processes;

• Be able to choose adequate models in practical socio-economic problems;

• Have skills in model construction and solving problems of time series analysis and stochastic processes.

4. Course Plan:

**Topic 1.Time series: main properties**

A rationale for time series modeling. Time series as a discrete stochastic process. Main characteristics of time series (expectations, moments, autocovariation, (partial) autocorrelation, correlograms). Stationarity and nonstationarity. Stationarity as the main characteristic of stochastic component of time series. Lagoperator.Testing for stationarity: graphical techniques and the formal unit root tests. (Augmented) Dickey-Fuller tests. Other tests of nonstationarity.

**Topic 2.Autoregressive-moving average ARMA (p,q) models.**

Moving average modelsMA(q). Condition of invertibility. Autoregressive models AR(p). Yule-Walker equations. Stationarity conditions. Autoregressive-moving average ARMA(p; q) models.

**Topic 3.Estimation of ARMA (p,q) models. Forecasting.**

Box-Jenkins methodology. Identification, estimation and testing of ARMA(p; q) models. Trends and seasonality. Goodness of fit in time series models. Information criteria. Forecasting with ARMA(p; q)models. Properties of forecasts.

**Topic 4.Modeling volatility using ARCH and GARCH.**

Time-varying volatility. The notion of conditional volatility. Properties, diagnostics, and estimation of GARCH models.

**Topic 5.Vector autoregressive models.Causality.**

Vector autoregressive models. The notion of causality. Granger causality. Instantaneous causality.

**Topic6.Stochastic processes. Basic definitions.**

Definition of stochastic process. Finite-dimensional distributions of stochastic processes.Basic properties of stochastic processes. Poisson Processes. Gaussian Processes; Standard Wiener process (Brownian Motion). Stationarity: strong, weak. Ergodicity. Filtration problem.

**Topic7.Markovchains.**

Markov processes as generalizations of IID variables and of deterministic dynamical systems. The Markov property and the strong Markov property. Classifications of States of Markov chain. Ergodic Markov chain. Limiting distribution of Markov chain.Gambling.

**Topic 8.Discrete-time Markov chain.**

A series of events. Chapman-Kolmogorov Equations. Ergodicproperties of homogeneous Markov chains. Birth and Death Processes. Queueing theory.

5. Reading List

a. Required

1. Brockwell, P.J., and R.A. Davis, 2003, Introduction to Time Series and Forecasting,Springer Publ., 2nd ed.
2. Mills, T.C. and R.N. Markellos, 2008, The Econometric Modelling of FinancialTime Series, Cambridge University Press, 3rd ed.
3. Enders, W., 2003, Applied Econometric Time Series, Wiley Publ., 2nd ed.
4. Hamilton J.D., 1994, Time Series Analysis, Princeton University Press.

b. Optional

1. Box, G.E.P., G.M. Jenkins, and G.C. Reinsel, 2008, Time Series Analysis: Forecasting and Control (Wiley Series in Probability and Statistics), Wiley Publ., 4th ed. – 784 pages. ISBN: 978-0-470-27284-8.
2. Lütkepohl, H., &Krätzig, M., 2004, Applied time series econometrics. Cambridge University Press.
3. Patterson, K.D., 2000, Introduction to Applied Econometrics: A Time Series Approach, Palgrave Macmillan.
4. Brockwell, P.J., and R.A. Davis, 2003, Introduction to Time Series and Forecasting,Springer Publ., 2nd ed.
5. Box, G.E.P., Jenkins G.M. and G.C. Reinsel, 2008, Time Series Analysis: Forecastingand Control (Wiley Series in Probability and Statistics), Wiley Publ., 4th ed.
6. Maddala, G.S. and In-Moo Kim, 1999, Unit Roots, Cointegration, and Structural Changes (Themes in Modern Econometrics), Cambridge University Press.
7. Banerjee, A., J. Dolado, J.W. Galbraith, and D.F. Hendry, 1993, Co-integration, error-correction, and the econometric analysis of non -stationary data, N.Y., OxfordUniv. Press
8. Harvey, A.C., 1993, Time Series Models, Harvester Wheatsheaf, 2nd ed.

6. Grading System

30% homework + 30% mid-term exam + 40% final exam

7. Guidelines for Knowledge Assessment

**Mid-term exam (approximate version)**

**1**. A stochastic sequence $ξ\_{n}$, $n\in Z$, satisfies a stationary second-order autoregression equation with known parameters. There are observations $ξ\_{50},ξ\_{51},ξ\_{52}$. Construct the best (in the mean square sense) forecast for the $ξ\_{53}$.

**2.** Put by definition $ξ\left(t\right)=\left\{\begin{array}{c}1, \&t<τ\\-1, \&t\geq τ\end{array}\right., t\geq 0, τ\~E\left(α\right).$

Find one-dimensional and two-dimensional distributions of the process $ξ\left(t\right)$.

**3.** Assume that X and $Y$ are independent random variables, X has a normal distribution with mean 0 and variance 1, Y has an uniform distribution on  Find the covariance functions of the stochastic process $ξ\left(t\right)=cos$*(t+Y), -*$\infty <t<\infty .$ Is the stochastic process $ξ\left(t\right)$ a stationary process?

**4.** Let $W(t)$ be a standard Wiener process. You have two observations of this process at points $t\_{1}$ and $t\_{2}$. Give the best (in the mean square sense) estimate of $W(t)$ at a point $t=t\_{3}$*,* $0<t\_{1}<t\_{2}<t\_{3}$. Find the mean square error $Δ=E(W(t\_{3})-\hat{W}(t\_{3}))^{2}$, of this estimate.

**Final exam (approximate version)**

1. The values of the sample autocorrelation function ACF (k) and the values of the sample partial autocorrelation function PACF (k), k=1,...,12. of the observed time series $X\left(t\right)$, t=1,...,225 are given in the table:

|  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| k | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| ACF | -0,757 | 0,502 | -0,31 | 0,222 | -0,183 | 0.156 | -0,135 | 0,124 | -0,117 | 0,014 | -0,110 | 0,096 |
| PACF | -0,757 | -0,166 | 0,019 | 0,089 | -0,027 | 0,005 | 0,071 | -0,006 | 0,013 | 0,004 | -0,007 | 0,004 |

Make identification of this time series. What further steps will you take to build the statistical model of the $X\left(t\right)$ series?

1. There are 100 observations of the time series $X\_{t}$. The estimates and the mean-square deviations of the estimates of the parameters of the model $X\_{t}=μ+βt+αX\_{t-1}++θ∆X\_{t-1}+ϵ\_{t} $were computed by the least squares method: The results of the estimation are given in the table:

|  |  |  |  |
| --- | --- | --- | --- |
| $$μ$$ | $$β$$ | $$α$$ | $$θ$$ |
| 53,02 (13,18) | 1,2 (0,24) | 0,751 (0,061) | 0,41(0,1) |

Lags of higher order are not included in the model, because the coefficients at lags of a higher order are considered insignificant. Find out if this series is a random walk.

1. There are 500 observations of the time series $X\_{t}$ By the method of least squares, estimates and mean-square deviations of the estimates of the parameters of the following models were calculated:$ X\_{t}=μ+βt+αX\_{t-1}+ϵ\_{t}$; $∆X\_{t}= μ+βt+ϵ\_{t}$; $X\_{t}=μ+αX\_{t-1}+ϵ\_{t}$, $∆X\_{t}= μ+ϵ\_{t}$ and $X\_{t}=αX\_{t-1}+ϵ\_{t}$. According to White's test, Breusch-Godfrey test and Jarque-Bera test, the residuals of each model are recognized as homoscedastic, uncorrelated and Gaussian, respectively. The results of the evaluation are given in the table:

|  |  |  |  |
| --- | --- | --- | --- |
| Model | $$μ$$ | $$β$$ | $$α$$ |
| 1 | 0,124(0,05) | 0,00025(0,00011 | 0,97(0,013) |
| 2 | 0,029(0,013) | 0,0007(0,0006) |  |
| 3 | 0,09(0,05) |  | 0,94(0,026) |
| 4 | 0,08(0,06) |  |  |
| 5 |  |  | 0,95 (0,019) |

Can you assume that the series $X\_{t}$ is a DSP series? The answer is justified.

1. A matrix of transition probabilities for a homogeneous Markov chain is given
 $\left(\begin{matrix}0.5&0.5&0\\0.5&0&0.5\\0&1&0\end{matrix}\right)$.

Is the chain ergodic? Find all stationary distributions of the chain.

1. The queuing system consists of two identical service channels with service rates μ. The stream of applications is the simplest Poisson flow with intensity λ. It is required: 1) to construct a stochastic graph of the service process; 2) write the Kolmogorov equations for calculating the probabilities of the states of the process; 3) find the stationary distribution of the process; 4) calculate the probability of idle time and the probability of full occupancy of channels in the case when λ = 2μ.

**Exam paper questions**

**Topic 1.**

1. Give the definition of a stochastic process.
2. Give the definition of a time series.
3. Give the definition of a realizations of the stochastic process.
4. Give the definition of finite-dimensional distribution functions of stochastic process.
5. What are the main characteristics of stochastic processes you know?
6. Formulate and prove the basic properties of a covariance function.

**Topic 2.**

1. Give the definition of a strictly stationary stochastic process.
2. Give the definition of a weakly stationary stochastic process.
3. What is the relationship between weak and strict stationarity?
4. Give the definition of a Gaussian stochastic process.
5. Give the definition of the Wiener process (Brownian Motion).
6. What is the relationship between random walk and Brownian motion?
7. Write n-dimensional density function of a Gaussian stochastic process.

**Topic 3.**

1. Give the definition of a discrete white noise
2. What time series are called linear stochastic processes?
3. What basic models of stationary time series do you know?
4. What are the conditions of the invertibility of a moving average process?
5. Find autocorrelation function of a moving average MA(q) process.
6. Find autocorrelation function of AR(1) process
7. What are the causes of nonlinear models?
8. Describe the nonlinear stationary models Autoregressive Conditional Heteroskedasticity (ARCH(p)) and Generalised Autoregressive Conditional Heteroskedasticity (GARCH(p;q))
9. What is called volatility of time series?

**Topic 4.**

1. Give the definition of an optimal in the mean square sense predictor.
2. What is called a mean square error of the predictor?
3. Specify the explicit form of the best(in the mean square sense) predictor.
4. Describe the procedure for constructing the best forecast for a Gaussian process.

**Topic 5.**

1. What are the statistical properties of the sample autocorrelation function (ACF)?
2. What are the statistical properties of the sample partial autocorrelation function (PACF)?
3. What is the asymptotic distribution of the sample autocorrelation function (ACF) for white noise?
4. What is the asymptotic distribution of the sample autocorrelation function (ACF) of a moving average MA(q) process?
5. What is the asymptotic distribution of the sample partial autocorrelation function (PACF) of an autoregressive AR(p) process?
6. Describe methods for estimating the parameters of the autoregression equation.
7. Describe the procedure for determining the order of the model of the autoregression process.
8. What criteria do you know about the adequacy of the selected ARMA model?
9. Describe the procedure for checking the Gaussianity of the residuals of the ARMA model.

**Topic 6.**

1. Describe Box-Jenkins methodology.
2. Describe the main stages of the information of the non-stationary time series to the stationary one.
3. What methods of estimating and excluding trend and seasonal components are known to you? Show on examples.
4. What are the main differences between non-stationary series such as TSP and DSP.
5. Describe Dickey-Fuller test and Augmented Dickey-Fuller tests.
6. Describe the Dolado-Jenkinson-Sosvilla-Rivero procedure. What is this procedure for?

**Topic 7.**

1. Specify vector autoregressive models
2. How to identify spurious regression?
3. What time series are called cointegrated?
4. How to identify the cointegration of two series?
5. What is Granger’s causality?

**Topic 8.**

1. What is the Markov chain?
2. Which chain is called homogeneous?
3. What states are called essential, communicating, aperiodic?
4. State the ergodic theorem.
5. How can one find the stationary distribution of a homogeneous Markov chain if it exists?

**Topic 9.**

1. Give the definition of the Poisson stochastic process
2. Which stream of events is called the simple Poisson stream?
3. Define the Markov property for a CS with continuous time and a finite set of states.
4. Write down the Kolmogorov algebraic equation system for a homogeneous Markov process.
5. Determine the intensity of the transition from state to state.
6. What kind of stochastic graph has the birth-death process?

8. Methods of Instruction

The discipline is delivered through lectures seminars, including computer classes.

9. Special Equipment and Software Support (if required): Computer classes